

Are There Top Quarks in Superdense Hybrid Stars?

M. Prisznyák, B. Lukács and P. Lévai

Institute of Nuclear and Particle Physics, Theor. Dep.

Budapest, H-1525 P.O.B. 49

E-mail: [surname]@rmki.kfki.hu

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Abstract

It is examined whether primordial top quarks from the Big Bang could survive under natural conditions till present days. Hybrid quark-neutron stars, with dense enough core to maintain the existence of top quarks, is discussed. In this paper only final states are investigated. Zero temperature quark-lepton matter is modelled as a gas of non-interacting particles. Local charge neutrality has been assumed in order to avoid volume interactions which would not make thermodynamical treatment possible.

I. INTRODUCTION

With the claimed discovery of the top quark at FERMILAB [1] the quark family has become complete and masses of all quarks are known. Now we are in the position to answer the question of occurrence of heavy quarks, including the top, in Nature. Such a scenario may belong to cosmology, more exactly the physics of Big Bang or to astrophysics, namely superdense quark stars. They may have dense enough cores to support the existence of heavy quarks and our aim is to investigate the necessary conditions for the existence and stability of these stellar objects.

According to the standard theories of stellar evolution, the final state of massive stars can be compact, dense stellar objects, neutron stars, with radius of 10 km and mass of $0.1-2 M_{\odot}$. Since the redshift parameter is about 0.2 on the surface of these objects one must use general relativity. For a non-rotating, static stellar object the Tolman-Oppenheimer-Volkoff [2,3] equation expresses the condition of hydrostatic equilibrium and connects nuclear physics, through the equation of state (EOS), with astrophysics, through gravitation. Since equilibrium configurations exist for *any* central densities, actually even for infinite central density as well, a quark core may support hadron matter in neutron stars. However stability against oscillations still must be checked.

It is a generally held belief in the physics community that very dense nuclear matter ($\rho \geq 5 - 6 \rho_{nuc}$) actually appears in the form of quark-gluon plasma (QGP), i.e. the physical degrees of freedom are no longer hadron particles but quarks and gluons.

We assume that locally the QGP matter is electrically neutral. Without this assumption we would have a volume self-interaction, so a non-extensive energy variable for our system and no thermodynamic approach could be applied. Thenceforth one could not tell either what different phases exist inside the star because to obtain phase boundaries one needs thermodynamics.

In Section II we discuss whether primordial top quarks could survive from the Big Bang. Section III is about the confinement transition of the Universe. Section IV deals with hybrid stars. Section IV C addresses the question of stability of these stellar objects.

We note that this paper is only a preliminary report of an on-going research work. At some points serious simplifying assumptions were used to make the calculations possible. Our main goal was to find the top peak on the mass-density plot if it exists at all. A future paper will rediscuss some simplifying assumptions.

II. PRIMORDIAL TOP QUARKS FROM BIG BANG?

Top quarks must have been present in the very early Universe in two ways. Today's observations suggest a positive total baryon or quark number in our Universe. At temperatures higher than $\approx (m_t - m_c)c^2$ a substantial part of this total quark number must have been in t quarks. True enough, we do not know when the present baryon number was created. Besides an originally matter-antimatter-asymmetric Universe, which is a not too attractive possibility, at least three different ideas have been seriously discussed. Namely, net baryon number may have been generated

- in a non-equilibrium situation just after the GUT symmetry breaking, i.e. at cca. 10^{-35} s and 10^{14} GeV temperature [4]
- during the electroweak transition ($t \approx 10^{-10}$ s, $T \approx 1000$ GeV) via anomalous processes through e.g. sphalerons [5]
- from lepton excess produced in an assumed SUSY transition, by anomalous processes before the electroweak transition [4].

Therefore the terminus ante quem for $B > 0$ seems to be 100 picosec. However, t quarks were present even without a quark excess, in the form of $t\bar{t}$ pairs. Such pairs are present in substantial quantity at $T \geq m_t c^2$. Both conditions hold practically at the same temperature, $T \geq 175$ GeV. To estimate characteristic times, let us take a radiation-dominated $k = 0$ Friedmann universe,

$$ds^2 = dt^2 - R^2(t)\{dx^2 + x^2 d^2\Omega\} \quad (2.1)$$

$$\varepsilon = 3P = (N\pi^2/30)T^4 \quad (2.2)$$

where N is the effective number of helicity degrees of freedom, 1 for each light boson and 7/8 for each light fermion pair. In the GUT symmetric phase $N \approx 162$, and just before the breakdown of electroweak symmetry $N \approx 10$. The solution of Eqs.(2.1-2.2) is

$$T = \sqrt[4]{\frac{3}{32\pi} \frac{30}{N\pi^2}} \sqrt{\hbar E_{Pl}/t} \quad (2.3)$$

where the E_{Pl} Planck energy is $1.22 \cdot 10^{19}$ GeV. Hence $T \approx 100$ GeV at $t \approx 100$ picosec. Therefore $t\bar{t}$ pairs vanished at 100 picosec, and excess tops, if already existed then, started to decay into charm and up quarks. The decay process is driven by weak interaction, therefore at this energy difference the lifetime is definitely shorter than 100 picosec. Consequently top quarks practically vanished from the Universe not later than 100 picosec = 10^{-10} s after the Beginning.

III. TOP QUARKS IN THE CONFINEMENT TRANSITION

Bottom quarks start to annihilate and decay at $T \approx 4.5$ GeV, which corresponds to cca. 10 nanosec. For a review of the early history of the Universe see e.g. [6]. Decay times are much shorter, so they vanish practically instantaneously. The same happens with the c quarks at 100 nanosec. The decay/annihilation begins for s quarks, too, at several microsecs, but that process is interrupted at 7 microsecs by a first order confinement transition [7]. In this transition 3-quark groups form baryons and quark pairs form mesons. The fine details may depend on unevaluated details of nonperturbative QCD; anyway, serious density differences can be expected during the transition which lasts between cca. 7 and 12 microsecs. One possible result is the formation of dense quark blobs. The later fate of these dense blobs is under debates; some authors claim that the so-called symmetric quark matter (equal weights of u, d and s) may be stable against hadron formation [8]. But the fate cannot be calculated without knowing the characteristic sizes of the blobs. The transition starts

from nucleation cores of the new phases. The evolution is uncorrelated at distances greater than ct , therefore at 7 microsecs the radius of the emerging blobs will be in the range of some kilometers. Consequently the blobs will resemble neutron stars of the order of solar mass. As it is well known, a neutron star is a stationary object sustained in its own gravity, and density inhomogenities inside can be large. Strictly speaking the central density of a neutron star has no upper limit; in all calculations equilibrium configurations exist for even infinite central density [9]. (Stability against radial oscillations is another matter and will be studied in due course.) So, the phase transition may regenerate the heavy quarks vanishing earlier. (Similarly can the gravitational collapse do so, resulting in neutron or quark stars at much later times.) The calculation of the equilibrium states needs full general relativity and the equation of state of relativistic QCD plasma.

IV. TOP QUARKS IN HYBRID STARS

A. The Tolman-Oppenheimer-Volkoff equation of general relativity

In general relativity theory there always exists a locally free-falling frame in which the laws of special relativity govern physics. For a uniform fluid at rest in this frame the $T_{\mu\nu}$ energy-momentum tensor defines what we mean by pressure and energy density. Once $T_{\mu\nu}$ is known in a covariant form in this frame it serves as a source term in the Einstein equation of GTR.

Let us restrict ourselves to static spherical objects. Then we have 4 Killing vectors in the space-time, of which 3 are space-like, act on 2 dimensional transitivity surface and commute as $SO(3)$; the fourth one is timelike and commutes trivially with all of the previous three [10]. Then, up to coordinate transformations, the most general possible line element is of form

$$ds^2 = \exp(2\nu(r))dt^2 - \exp(2\lambda(r))dr^2 - r^2d\Omega \quad (4.1)$$

Now, the curvature is governed via the Einstein equation

$$R_{ik} - \frac{1}{2}g_{ik}R = -8\pi GT_{ik} \quad (4.2)$$

where R_{ik} is the Ricci tensor and T_{ik} is the energy-momentum tensor. Henceforth $c = 1$. For a fluid the latter one has the form $T_{ik} = \varepsilon u_i u_k - P(g_{ik} - u_i u_k)$. Here ε is the energy density and P is the pressure. Both can be calculated from the equation of state. The velocity field of the fluid is in our case purely temporal, and of unit length.

$$u_i = (1, 0, 0, 0) \quad (4.3)$$

For the present symmetries eq. (4.2) possesses three nontrivial and algebraically independent components, say the 00, 11 and 22 ones. However the velocity field (4.3) is a solution of the equation of motion, following from the automatically vanishing divergence of the left hand side of eq. (4.2). So now two components will suffice. We can write them as follows:

$$8\pi GT_1^1 = 8\pi GP = -e^{-\lambda}\left(\frac{\nu}{r} + \frac{1}{r^2}\right) + \frac{1}{r^2} \quad (4.4)$$

$$8\pi GT_2^2 = 8\pi GP = -\frac{1}{2}e^{-\lambda}\left(\nu'' + \frac{1}{2}\nu'^2 + \frac{\nu' - \lambda'}{r} + \frac{\nu'\lambda'}{2}\right) \quad (4.5)$$

Now let us write, simply as a *definition*:

$$\exp(-2\lambda(r)) \equiv 1 - 2m(r)/r \quad (4.6)$$

Then an integro-differential equation is obtained for $m(r)$, which we write here as

$$\frac{dP}{dr} = -G^{\frac{(\varepsilon+P)(m(r)+4\pi Pr^3)}{r(r-2m(r))}} \quad (4.7)$$

$$m(r) = 4\pi \int_0^r \varepsilon(\tilde{r}) \tilde{r}^2 d\tilde{r} \quad (4.8)$$

and there remains a quadrature for the function $\nu(r)$, which is trivial and will be ignored here. In the last equation we exploited a regularity relation $\lim_{r \rightarrow 0} m(r) = 0$. Eq. (4.7) is the Tolman-Oppenheimer-Volkoff (TOV) equation. If the equation of state is fixed as a function $P = P(\varepsilon)$ or in any equivalent parametric form, then to any central density $\varepsilon(0) = \varepsilon_0$ a unique solution can be obtained.

B. Phase transition

In our environment the low density state of quarks is hadronic matter, i.e. a mixture of tightly bound qqq and $q\bar{q}$ clusters. It is not quite sure that this is the lowest energy state indeed; there are speculations [8] that the "symmetric strange matter", i.e. an equal weight mixture of u , d and s in plasma state, may be energetically preferred against neutron matter, and then the fact that nuclear matter remains in hadrons could be explained in terms of the macroscopic S charge of such a state. Then there may be a high potential barrier between the local $S = 0$ minimum of nuclear matter and the global minimum of $S = B/3$. The final answer is not yet known because the calculations are rather difficult for dilute quark plasma with QCD being there nonperturbative. Now, observe that the "potential barrier" argumentation may still be valid in situations where hadronic matter transforms into quarks in a compression process. For the primordial quarks forming a quark star between 7 and 12 microsecs the argument is clearly irrelevant. Namely, at that time s quarks still were abundant; star-sized blobs of the symmetric (uds) plasma can be formed by using up the existing s quarks. (This is roughly the suspected scenario of "strange nugget" formation [11].) Therefore it is better not to make preconceptions about the low-density crust of the stars under investigation. However the conditions for phase transition are clear. Hadrons h_α are built up from quarks q_i with stoichiometric numbers c_α^i :

$$\sum_r c_\alpha^r q_r \rightarrow h_\alpha \quad (4.9)$$

Then phase equilibrium holds if

$$\sum_r c_\alpha^r \mu_r = \mu_\alpha \quad (4.10)$$

$$p_{QGP} = p_H \quad (4.11)$$

Now, we start from inside where the matter is a quark plasma with no doubts. If, going outward, somewhere Eqs. (4.10-4.11) holds then there a phase transition happens and the TOV equation is to be continued with the equation of state of the new phase.

Since now there are two different quark chemical potentials (see Subsection IV D) the actual phase structure may be quite complicated (for a discussion see [12].) However it seems that, if hadronic phase exists, it is dominated by neutrons; e.g. Ref. [9] got 7/8 part n vs. 1/8 part p . Then for a first approximation the phase transition is

$$3q \rightarrow n$$

where one quark is u , c or t , two are d , s or b ; and Conds. (4.10-4.11) read as

$$\mu_u + 2\mu_d \approx \mu_n \quad (4.12)$$

$$p(q) \approx p(n) \quad (4.13)$$

This will be our condition to switch to nuclear equation of state.

C. The stability of configurations

An equilibrium configuration may still be unstable against oscillations. The sufficient and necessary conditions of stability against oscillations can be found in the B Appendix of Ref. [9]. In that work a series of theorems has been proven for those who are interested only in the question of the stability and not in oscillation frequencies.

The oscillation frequency squares form a series of disjoint values $\Omega_i^2 = \Omega_i^2(\varepsilon_0)$ where ε_0 is the central energy density of the configuration. A mode becomes unstable where its Ω_i^2 becomes zero from above. A mode can change its stability only in points where $M(\varepsilon_0)$ has an extremum. Ignoring possible inflexion points, one gets the scheme that (in parentheses the shorthand terms referring to directions of turns on an $R(M)$ diagram) for maximum

- with $R(\varepsilon_0)$ increasing (left turn on the right hand side) : the last unstable mode becomes stable;
- with $R(\varepsilon_0)$ decreasing (right turn on the right hand side) : the first stable mode becomes unstable;

for minimum

- with $R(\varepsilon_0)$ increasing (right turn on the left hand side) : the first stable mode becomes unstable;
- with $R(\varepsilon_0)$ decreasing (left turn on the left hand side) : the last unstable mode becomes stable;

(See also Ref. [13].) Since until now all calculations for the neutron star peak gave stability on the slope upwards, stability of all peaks can be read off from the $M(\mu)$ and $R(\mu)$ curves if they are sufficiently reliable in details.

D. The equation of state

At first we assume that the core of the superdense quark star can be described as non-interacting Fermi gas of quarks and leptons at zero temperature. Having no internal energy production, these stellar objects will quickly cool to near absolute zero [14].

Thus the degrees of freedom taken into account are fermionic. The rôle of other bosonic degrees of freedom (e.g. quasi-particles made up of interacting fermions) will be discussed in a subsequent work. At zero temperature we need not deal with antiparticles either. The effect of nonperturbative interaction among quarks will be simulated by introducing the bag constant, B , which is important at lower densities to determine the interface between the quark core and the hadronic surface layer. This value of $B = 260$ MeV is derived from phenomenological high energy physics and may not be quite suitable for matter at these densities but this is the best value available now.

Since on cosmic time scale the weak interactions will assure chemical equilibrium, one can introduce chemical potentials for all particle species and linear equations will hold among them. The chemical equilibrium is established by the following one- W exchange processes:

$$\begin{aligned} s + u &\leftrightarrow d + u \\ c + s &\leftrightarrow u + s \\ b + c &\leftrightarrow s + c \\ t + b &\leftrightarrow c + b \end{aligned} \tag{4.14}$$

These equations imply the following relations for the chemical potentials of the differently charged members of the quark doublets:

$$\mu_u = \mu_c = \mu_t \equiv \mu \tag{4.15}$$

$$\mu_d = \mu_s = \mu_b \equiv \tilde{\mu} \tag{4.16}$$

However, upper heavier quarks decay into lower lighter ones via the reaction

$$Q \rightarrow q + W \rightarrow q + l + \bar{\nu}_l \tag{4.17}$$

(Top quarks decay almost exclusively via $t \rightarrow b + W$.) The lepton conservation is insignificant in our calculation because the neutrinos have negligible partial pressure and they can escape soon after their production. Thus we keep only the chemical potential for electrons, muons and taus, which must be equal: $\mu_e = \mu_\mu = \mu_\tau \equiv \mu_l$. Moreover, the decay equation (4.17) couples the chemical potential of the quark doublets:

$$\tilde{\mu} = \mu + \mu_l \tag{4.18}$$

Thus we have only two variables left. If we assume local electric charge neutrality then there will remain only one independent variable, say, μ , to describe the quark-lepton matter at zero temperature.

Now let us summarize the necessary expressions for the density, energy density and pressure of the quark-lepton matter. For a particle of given type the thermodynamic quantities are the following (with $\hbar = c = 1$):

$$n_i = \frac{g_i}{(2\pi)^3} \int_{m_i}^{\mu_i} d^3k = \frac{g_i}{6\pi^2} (\mu_i^2 - m_i^2)^{3/2} \quad (4.19)$$

$$\varepsilon_i = \frac{g_i}{(2\pi)^3} \int_{m_i}^{\mu_i} \varepsilon_i(k) d^3k = \frac{g_i}{8\pi^2} [\mu_i (\mu_i^2 - m_i^2)^{1/2} (\mu_i^2 - \frac{1}{2} m_i^2) + \frac{m_i^4}{2} \log \frac{m_i}{\mu_i + (\mu_i^2 - m_i^2)^{1/2}}] \quad (4.20)$$

$$p_i = n_i \frac{de_i}{dn_i} - e_i = \frac{g_i}{24\pi^2} [\mu_i (\mu_i^2 - m_i^2)^{1/2} (\mu_i^2 - \frac{5}{2} m_i^2) - \frac{3}{2} m_i^4 \log \frac{m_i}{\mu_i + (\mu_i^2 - m_i^2)^{1/2}}] \quad (4.21)$$

Here the one-particle energy is $\varepsilon_i = \sqrt{k^2 + m_i^2}$, $g_i = 6$ is the degeneracy for quarks ($i = u, d, s, c, b, t$) and $g_i = 2$ is the degeneracy for leptons ($i = e^-, \mu^-, \tau^-$).

The total energy and pressure of the quark-lepton matter is the following:

$$\varepsilon = \sum_i e_i \Theta(\mu_i^2 - m_i^2) + B^4 \quad (4.22)$$

$$p = \sum_i p_i \Theta(\mu_i^2 - m_i^2) - B^4 \quad (4.23)$$

$\Theta(x)$ is the Heaviside function, B is the phenomenological bag constant and the chemical potential can be determined from Eqns. (4.15,4.16,4.18) as the function of μ and μ_l .

Plugging in Eqs. (4.22,4.23) into Eq. (4.7) and using the mass continuity condition (4.8) we still need another equation to connect our independent variables μ and μ_l .

The condition of local charge neutrality yields the necessary expression to determine the quark chemical potential μ dependence of leptonic chemical potential μ_l :

$$\sum_i n_i q_i \Theta(\mu_i^2 - m_i^2) \text{sgn } \mu_i = 0 \quad (4.24)$$

where $q_i = 2/3$ is the electric charge for ($i = u, c, t$), $q_i = -1/3$ is the charge for ($i = d, s, b$) and $q_i = -1$ for leptons ($i = e^-, \mu^-, \tau^-$).

E. The system of differential equations to be integrated

We have Eqs. (4.7,4.8,4.24) to be solved numerically in order to obtain an equilibrium configuration of a dense stellar object.

This system of equations is governed by μ of Eq.(4.15) which determines the (central) energy density, i.e. the source of gravitation. The technical details of some numerical problems, due to the shoulder in the density profile, are discussed in Appendix B.

The phase transition into hadronic matter occurs where

$$p_H = p_{QGP} \quad (4.25)$$

$$\frac{d\varepsilon}{dn_H} = \mu_H = 2\mu_d + \mu_u = 3\mu_u + 2\mu_e$$

We neglect the pressure coming from the leptons at this stage.

From this point on we use a hadronic EOS [15] with parameters $c_v^2 = 195.7$ and $c_s^2 = 266.9$ until the surface of the stellar object is reached, i.e. where $p_H = 0$. This $T = 0$ EOS exhibits nuclear saturation and has been derived from a relativistic mean field theory incorporating neutral scalar and vector meson fields. The two dimensionless coupling constants of the

theory were matched to the binding energy and density of infinite nuclear matter in order to obtain results at all densities.

Since we are discussing cold matter, the equations for the energy density and pressure are equivalent to knowing $T_{\mu\nu}$. This treatment is fully relativistic.

We checked our model against a simple EOS, quadratic in the nuclear density, as well which yielded similar results.

F. The shortcomings of our approach

Let us think over what the main deficiencies in our treatment may be.

As for the quark matter, we used a phenomenological bag model with a bag constant and current quark masses as parameters. The bag constant B , which parametrizes the difference of the exact and perturbative vacua (thus actually accounts for interactions among quarks) is measured only in hadron spectroscopy at low energies, so it is rather uncertain and this is inherited by both ε and p . Similarly, the EOS was an asymptotically free quark plasma which is clearly a simplification. However to this point we can note that at 174 GeV the quark plasma is expected to be fairly perturbative.

At the t peak we are already at energies well in the order of magnitude of the Weinberg-Salam mixing of electroweak interactions. While in itself this mixing may be of secondary importance for the EOS, a possible accompanying first order phase transition would not be insignificant since then a latent heat/compression energy of the order of magnitude $\frac{E_0^4}{(\hbar c)^3}$ would appear simulating a substantial another "bag constant". It is not clear yet if the Weinberg-Salam symmetry breaking transition is a first order transition or not. However no doubt that $m_W = 80$ GeV and $m_Z = 91$ GeV, well below m_t .

For hadron matter we used a Walecka-type EOS whose validity range is $\rho \geq 5 \cdot 10^{14} g \text{ cm}^{-3}$. Also, it definitely cannot be relevant below $\approx 10^{12} g \text{ cm}^{-3}$ where inverse β -decay sets in. However, as it turns out, the shell where our hadronic EOS breaks down carries only some hundredths of solar mass. Anyway, our aim was not to get accurate numbers for masses or radii but to qualitatively investigate a central density regime not explored as far as we know.

We neglected electron pressure in the hadronic matter which is significant at matter densities in the order of $\rho \approx 10^{13} g \text{ cm}^{-3}$ and could change our results for the radii.

We shortcutted the subtle question of phase transition by saying neutrons dominate. However cca. 10 % protons (and electrons to ensure charge neutrality) and probably other hadronic particles, up to a few per cents, like Δ 's and heavy mesons also contribute and they may change the phase structure. For a truly satisfactory discussion we would need a single EOS which contains, as limiting cases, both the hadronic and quark matter EOS. The less explicit (weaker first order or possibly second-order) the deconfinement phase transition is, the less acute this problem of supercompressing and overrarifying is [6].

V. CONCLUSION

Our results are summarized in a series of figures. The masses and radii of quark stars have been calculated at cca. 340 different central chemical potential values, and even then

numerical techniques were needed to smoothen the curves since at high central densities the inhomogeneities are substantial to cause numerical difficulties.

Fig. 1 displays the total mass of the equilibrium configurations vs. central chemical potential (monotonous with central energy density). The curve starts at the point where the quark plasma appears in the center.

Fig. 2 is the same but for radius. The curve is qualitatively similar to Fig. 1.

Fig. 3 composes the two earlier figures and depicts $R(M)$. By means of this curve one can directly apply the stability criteria of Section IV C. For increasing central density the curve starts in the upper right corner, where quark matter appears, and ends up in the middle, where top Fermi gas could exist in the very inside of a hybrid star.

Note that prior to this point the star is an ordinary neutron star with Walecka-type equation of state. (Some configurations are shown with dashed line.) There $M(\varepsilon_0)$ is on the rise and this upward slope was found to be stable. (see e.g. Ref. [9]). This turning point corresponds to the Landau-Oppenheimer-Volkoff point [16]

At the first turning point (point 1) on Fig. 3 R is growing at a maximal M . Consequently the mode of lowest frequency becomes unstable. The next extremal point is a mirror turn (turn to the right on the left side), therefore the lowest frequency stable mode becomes unstable again. This point corresponds to the Misner-Zapolsky point [17].

The form of the quark equation of state is qualitatively similar to the EOS of the Fermi gas of non-interacting neutrons of the early calculations and this may explain the qualitatively similar structure of our $R(M)$ curve to that of Ref. [9].

Beyond this point M is again increasing; we see the asymptotically damped oscillation of $M(\varepsilon_0)$ (or $M(\mu)$) as demonstrated in Ref. [9].

However not too far from the M-Z point c quarks appear in the core, later they become relativistic and, as usual, the $M(\varepsilon_0)$ ($M(\mu)$) curve stops to increase while reaching the relativistic Fermi regime. This fact puts point 3 to slightly beyond chemical potential 2 GeV. Hence another stable mode becomes unstable, similarly as found earlier for pure non-interacting neutron matter.

Fig. 4 shows that we lose still another stable mode at the next turning point (point 4). Comparison to Fig. 1 gives that this turning point is located at $\mu = 5000$ GeV, which corresponds to the appearance of b quarks beyond 4500 GeV. The minimum is extremely shallow and wide.

The next turning point (point 5), located at 192 GeV, restores one mode as for stability. The peak is very small ($0.01 M_\odot$ compared to the background) but rather expressed; it seems to belong to t quarks.

Since no new degrees of freedom may appear in the equation of state beyond t quarks, we stopped the calculation after this turning point. Thenceforth the $M(\varepsilon_0)$ curve must display the well-known asymptotically damped oscillation with increasing instability.

Fig. 5 shows the energy density profile at the t peak. Note the sudden drop in the 10 cm range which coincides with the disappearance of t quarks. The jump of the first order phase transition is at cca. 2 km.

Fig. 6 is the change of the two chemical potentials, as connected through charge neutrality, in a configuration at the t peak. The μ curve decreases smoothly and monotonously but μ_l shows a turning point at about 300 m. This surprising behaviour needs an explanation.

The u , d , c, s, e^- and μ^- number densities are shown in Fig. 7. (In this range $0 \leq \mu \leq 2000$ MeV, $n_\tau = 0$.) Now observe that descending with μ the electron density must drop substantially at 1500 MeV since there the c quarks vanish and there remain two negative light quark flavours with $-1/3 e$ charge each to compensate for the single positively charged one with $+2/3 e$. However n_{e^-} must not drop to 0 since n_s is slightly below n_u or n_d due to the higher s mass. Much later, at $\mu_u + \mu_e \approx 150$ MeV the s quarks vanish and then a great amount of electrons will be needed to balance the double charge of u over d . This way μ_{e^-} must slowly increase with decreasing μ_u after a minimum which happens to occur at $\mu = 1420$ MeV.

Ref. [18] calculated quark stars up to charm, but without assuming local neutrality and a hadronic mantle. They did find a separate s peak, but no peak for light quarks. However the assumptions of the two models were sufficiently different to give such different results.

We may conclude that, at least in the present approximation, no stable configuration has been found beyond the LOV point where the star is mainly a neutron star but with uds plasma in the centre. If our approximations are valid then no c , b or t quarks are expected to exist in the present Universe at least under natural conditions.

The present paper is only a first approximation to the problem. We listed the serious simplifications in Section IV F. Some points will be rediscussed in a future work. Note that e.g. at the starting point of Figs. 1 – 4 these calculations would need nonperturbative QCD results, practically unavailable now.

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APPENDIX A: HOW TO AVOID THE NUMERICAL DIFFICULTIES AT $R = 0$

Since Eq. (4.7) is singular at $r = 0$ one has to start integrating at a small positive \bar{r} . At high densities it is necessary to take into account the mass inside this sphere of radius \bar{r} . Therefore we applied a first-order expansion of Eq. (4.7). Then one can choose the starting point of the numerical integration at $r = \bar{r}$ where the initial (central) density (or the equivalent μ) drops to, say, 95 percents. This \bar{r} can be expressed as $\bar{r} = \sqrt{0.05\mu^4/C}$ where $C = 2\pi G(\varepsilon(r=0) + p(r=0))(\varepsilon(r=0)/3 + p(r=0))$.

APPENDIX B: HOW TO AVOID THE NUMERICAL DIFFICULTIES WITH THE SHARPLY DECREASING DENSITY PROFILE

Since at densities where top quarks appear one has numerical difficulties with the integration of Eqs. (4.7, 4.8, 4.24, 4.22, 4.23), we resorted to various expansions.

At infinite central energy density $\rho = Kr^{-2}$ is a solution to Eq. (4.7). One gets the value of $K = \frac{3c^2}{56\pi G}$ by comparing the powers in r .

We get a regular solution at $r = 0$ if we require that $\rho = \frac{K}{r^2}f(r)$ is a regular function where $f(r)$ is an unknown function we find by substituting into Eq. (4.7). This way the density profile around $r = 0$ is

$$\rho(r) = \rho_0 - \frac{32\pi G\rho_0^2}{3c^2}r^2 + \frac{7936\pi^2 G^2\rho_0^3}{3c^4}r^4 + \dots \quad (\text{B1})$$

We used an expansion up to 16th order that breaks down, at densities relevant to the study of top matter, within a few centimeters. From that radius we applied another expansion as follows.

Performing perturbation calculation around the $\rho = K/r^2$ solution (i.e. now we seek a solution of the form $\frac{K}{r^2}(1 + \epsilon(r))$ where ϵ is small) which belongs to the infinite central density, one obtains an expansion at $r = r_0$ where r_0 is the endpoint of the validity range of Eq.(B1):

$$\rho(r) = \frac{K}{r^2}(1 + a_1 r + 0.464286a_1^2 r^2 + \dots) \quad (\text{B2})$$

Here parameter a_1 remains free to allow to join expansion (B1) and expansion (B2) in a continuous way.

We designated an arbitrary density (say, which belongs to $\mu = 25$ GeV, so that the relation $P \approx \frac{1}{3c^2\rho}$ still holds) at which we continued the numerical solution of the set of equations.

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FIGURES

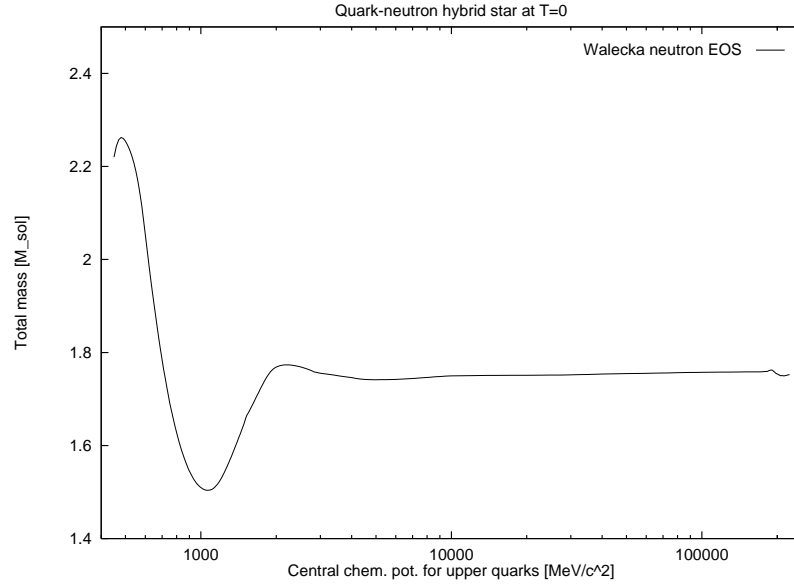


FIG. 1. Mass dependence of hybrid stars on the central chemical potential μ .

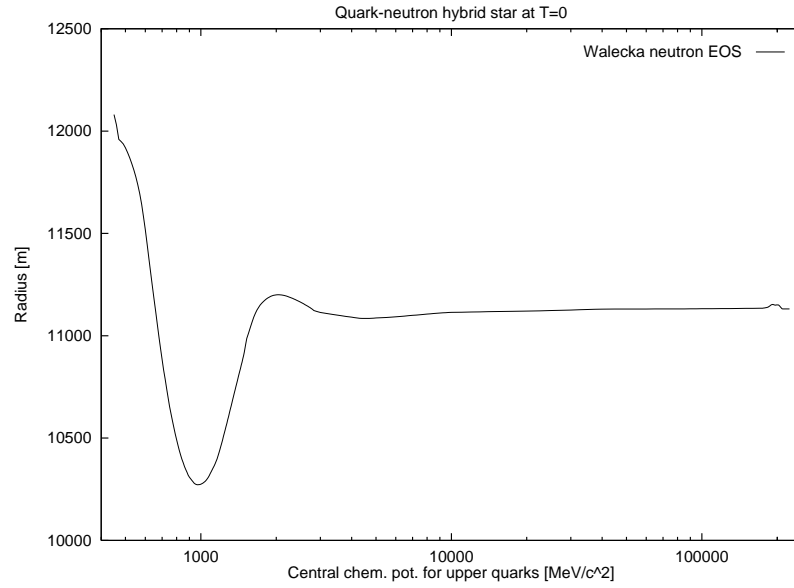


FIG. 2. Radius dependence of hybrid stars on the central chemical potential μ .

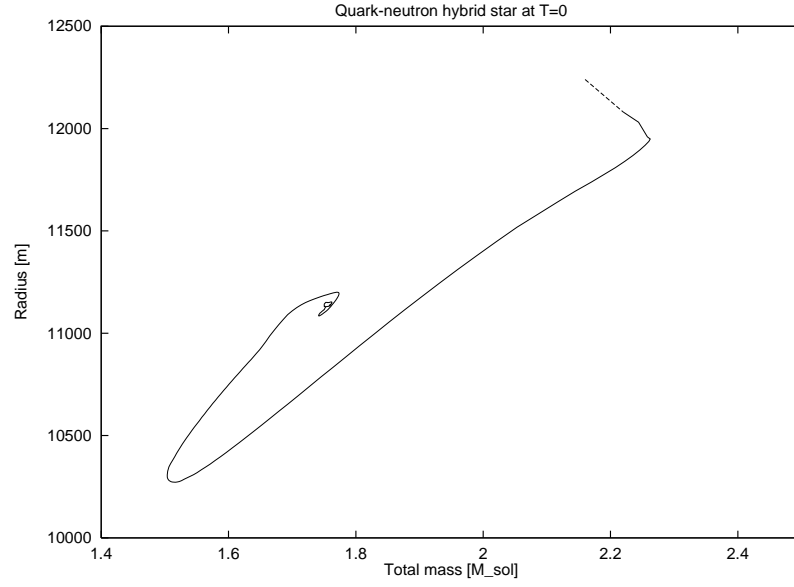


FIG. 3. Total mass vs. radius of hybrid stars.

FIG. 4. Total mass vs. radius in the vicinity of the top peak.

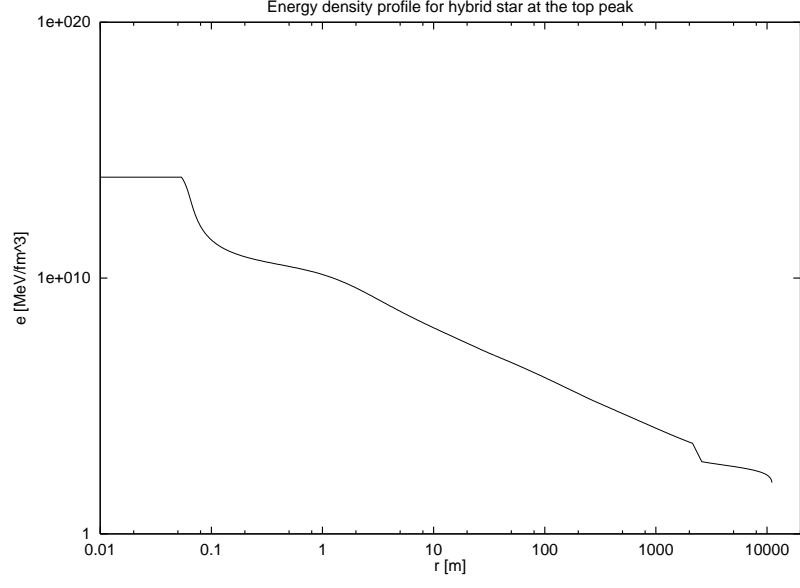


FIG. 5. Energy density vs. radius in the model hybrid star with central density at the the top peak.

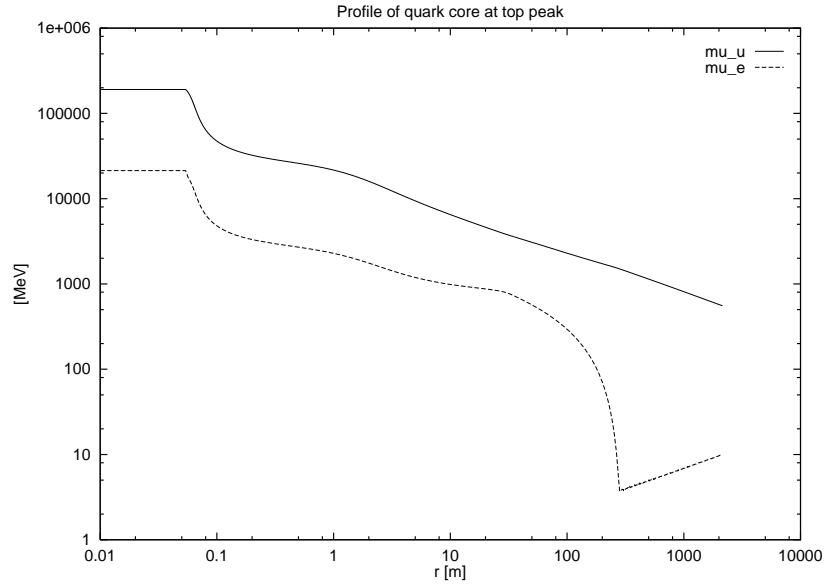


FIG. 6. Profile of the model hybrid star with central density corresponding to the top peak.

FIG. 7. Particle number densities vs. chemical potential μ .

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This figure "fig2-1.png" is available in "png" format from:

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